Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – Nov/Dec– 2017**

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| **Code :** | **15MA3009** | **Duration :** | **3hrs** |
| **Sub. Name :** | **FIELD THEORY** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | Let *R* be a ring. Then prove that *R[x]* forms a ring with respect to polynomial addition and multiplication | CO1 | 10 |
| b. | If *F* is a finite field and *α ≠ 0* and *β ≠ 0* are two elements of *F,* then prove that there exist *a* and *b* in *F* such that *1+αa2 + βb2 = 0.* | CO1 | 10 |
| (OR) | | | | |
| 2. |  | State and prove *Wederburn theorem* | CO1 | 20 |
| 3. | a. | If *f(x)* and *g(x)* are two non-zero elements of *F[x],* then prove that *deg(f(x)g(x)) = deg f(x) + deg g(x).* | CO2 | 10 |
| b. | Prove that any polynomial in *F[x]* can be written in the unique manner as a product of irreducible polynomials in *F[x]* | CO2 | 10 |
| (OR) | | | | |
| 4. | a. | Prove that the ideal *I = <p(x)>*in *F[x]* is a maximal ideal of *F[x]* if and only if *p(x)* is irreducible over *F.* | CO2 | 10 |
| b. | State and prove *Eisenstein criteria for polynomial*. | CO2 | 10 |
| 5. |  | If *K* is a finite extension field of *F* and *L* is a finite extension of *K*, then prove that *L* is a finite extension of *F* and *[L:F] = [L:K][K:F]* | CO2 | 20 |
| (OR) | | | | |
| 6. | a. | State and prove *Remainder Theorem* | CO2 | 10 |
|  | b. | Prove that a polynomial of degree n over a field *F* has atmost*n* roots in any extension field. | CO2 | 10 |
| 7. | a. | Find the degree of the following  i*. Q() over Q ii. Q() over Q iii. Q() over Q* iv. Prove that *Q() = Q()* | CO2 | 10 |
|  | b. | Find the deg of the extension of the splitting field of x3 – 2 ε Q[x] | CO2 | 10 |
| (OR) | | | | |
| 8. |  | Prove that *K* is a normal extension of *F* if and only if *K* is the splitting field of some polynomial over *F.* | CO3 | 20 |
|  | | **Compulsory**: |  |  |
| 9. |  | Let *f(x)* be a polynomial in  *F[x]* and *K* be its splitting field over *F*, *G(K, F)* be its Galois group for every subfield *T* of *K* which contains *F.* Let *G(K, T) = { ε G(K, F) / (t) = t for all t in T},* for every subgroup *H* of *G(K, F),* let *KH = {x ε K / (x) = x for all in H}.* Then prove that the association of *T* with *G(K, T)* set up a 1-1 correspondance of the set of subfields of *K* which contains *F* onto the subgroup of *G(K, F)* such that  *i. T = KG(K, T) ii. H = G(K, KH)* iii. *[K, T] = O(G(K, T))* iv. *T* is a normal extension of *F* if and only if *G(K, T)* is a normal subgroup of *G(K, F).* v. When *T* is a normal extension of *F, G(T, F)* is isomorphic to | CO3 | 20 |

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